

## NONISOTHERMAL FLOW OF NANOFUID IN GROUND HEAT ACCUMULATOR FOR DECENTRALIZED HEAT SUPPLY OF RURAL FACILITIES FOR VARIOUS PURPOSES

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**Abstract.** In rural areas, there is a question of heat supply to objects for various purposes that are located in areas remote from the infrastructure. The problem of using renewable energy sources and reducing CO<sub>2</sub> emissions is also relevant. Such objects with decentralized heat supply can be individual residential buildings, administrative, sports, cultural facilities, as well as greenhouses, livestock complexes, and storage facilities for agricultural products. To increase the efficiency of using solar heating, it is advisable to use heat accumulators. When using decentralized heat supply, seasonal heat accumulators provide greater efficiency and autonomy. A way to increase efficiency in such systems is to use nanofluids (propylene glycol/ethylene glycol-based nanofluid) as the heat carrier. These heat carriers are non-Newtonian fluids. Most industrially important heat carriers circulating in the round channels of soil accumulators at sufficiently high shear rates demonstrate high elasticity. In recent years, the processes of unsteady flow of elastic-viscous liquids in pipes and round channels of ground heat accumulators, which are caused by the action of various factors: time-varying pressure gradient, movement of layers, their general action, etc., have been intensively studied. The non-isothermal flow of an elastic-viscous fluid under the action of a pulsed pressure gradient in a round channel of a ground heat accumulator is considered. The rheological model is a nonlinear integral model with a memory function, which depends on the invariants of the strain rate tensor. Unsteady flow is considered and comprehensively studied; a system of equations is established that describes the flow of liquid with the specified rheological equation in the gap between two coaxial cylinders. The numerical analysis showed that the rheological and thermophysical properties of the fluid have a significant impact on the development of the pressure flow of an elastic-viscous fluid in a round channel.

**Keywords:** non-isothermal pressure flow, ground heat accumulator, nanofluid, coolant in the heat accumulator, non-Newtonian fluid.

### Introduction

In rural areas, there is a question of heat supply to objects for various purposes that are located in areas remote from the infrastructure. The problem of using renewable energy sources and reducing CO<sub>2</sub> emissions is also relevant. Such objects with decentralized heat supply can be individual residential buildings, administrative, sports, cultural facilities, as well as greenhouses, livestock complexes, and storage facilities for agricultural products. To increase the efficiency of using solar heating, it is advisable to use heat accumulators. When using decentralized heat supply, seasonal heat accumulators provide greater efficiency and autonomy. A way to increase efficiency in such systems is to use nanofluids (propylene glycol/ethylene glycol-based nanofluid) as the heat carrier. These heat carriers are non-Newtonian fluids [1; 2].

Most industrially important solutions and mixtures demonstrate high elasticity at sufficiently high shear rates (including in pressure flow processes in the annular channels of heat accumulators). Recently, the processes of unsteady flow of elastic-viscous liquids in pipes and (ring) channels of ground heat accumulators, which are caused by the action of various factors: time-varying pressure gradients, movement of boundaries, their general action, etc., have been intensively studied. But the non-isothermal flow of an elastic-viscous fluid under the influence of a pulsed applied pressure gradient, which is modeled by a nonlinear integral model with a memory function, depends on the invariants of the strain rate tensor, and is realized in the annular channels of ground heat accumulators. When studying the movements of rheologically complex liquids and mixtures in pipeline pipes, problems constantly arise with the “clogging” of the pipeline section with the mixture (which sticks, sharply narrows the pipeline cross-section, which significantly reduces the efficiency of pumping such liquids and mixtures by the pipeline, since it is necessary to install additional pumps. Such actions lead to a sharp increase in operating energy, and costs (especially when transporting rheologically complex liquids/mixtures over long distances). The heat exchangers of ground heat accumulators can have very long pipe lengths.

In [3-6], the flow of an elastic-viscous fluid was studied, which was initially at rest and arose under the influence of a pulsed pressure gradient. The three-constant Oldridge model [3-5], second-order fluid [5], and the linear integral model [6] were considered as rheological equations. The elastic properties of the fluid lead to the fact that the process of reaching a stationary regime becomes oscillating, and this

qualitatively distinguishes the behavior of such fluids from the behavior of inelastic fluids, for example, Newtonian power-law fluids [7-9], which are characterized by a gradual filling of the velocity profile and an increase in the fluid flow. Such results are in qualitative agreement with experimental data on measuring the flow rate of the polyethylene solution presented in [8].

Another option for the starting problem is to study the behavior of the liquid according to the flow rate specified at the initial moment of time [10-12]. The works [13-15] describe a general method of numerical solution of the problem of unsteady flow of viscous incompressible fluid in flat channels of arbitrary shape of heat exchangers. It should be noted that various aspects of convective heat and mass transfer of rheologically complex liquids are considered and studied in works [16-18].

**Materials and methods**

The purpose of this work is to substantiate a rheological model that adequately describes the non-isothermal flow of an elastic-viscous fluid in the annular channel of a ground heat accumulator under the influence of a pulsed pressure gradient. A nonlinear integral model with a memory function, depending on tensor invariants and circulation rates, is used as the rheological model. General form of the equation, with notations adopted in classical rheology [19-23]:

$$T(t) = \int_{-\infty}^t m((t-t'), \Pi_D(t')) \cdot \left( 1 + \frac{\varepsilon}{2} \cdot c^{-1}(t,t') + \frac{\varepsilon}{2} c(t,t') \right) dt',$$

$$m(t-t'; \Pi_D(t')) = \sum_k \frac{G_k}{\lambda_k} \cdot f_k(\Pi_D(t')) \cdot \exp \left\{ - \int_{t'}^t \frac{\varphi_k(\Pi_D(t''))}{\lambda_k} dt'' \right\}, \tag{1}$$

$$\Pi_D^2 = 2 tr D^2$$

When  $f_k = 1/(1 + 2\Pi_D(t') \cdot (\lambda'_k)^2)$  from (1) the Bird-Carreau model emerges;

with  $f_k = 1, \varphi_k = 1 + a \cdot \lambda_k \cdot (\Pi_D(t''))^{1/2}$  – Meister model;

with  $f_k = 1 + 2\lambda_k'^2 \cdot \Pi_D(t')^{1/2} / 1 + 2\lambda_k^2 \cdot \Pi_D(t')^{1/2}$ ,

$\varphi_k = [1 + |(2\lambda'_k \cdot \Pi_D(t''))^{1/2}|]^{3/2} / [1 + |(2\lambda_k \cdot \Pi_D(t''))^{1/2}|]^{1/2}$  – Macdonald-Bird-Carreau model.

In this work, it is proposed to use the equation system (1), which can be analytically solved, instead of Oldridge, Beard-Carreau and other models, which can be solved numerically only. Equation (1) after differentiation with respect to time can be represented as an equivalent system of first-order differential equations [24]. Let us consider one-dimensional unsteady flow. The system of equations that describe the flow of fluid with the rheological equation (1) in the gap between two coaxial cylinders ( $R_1$ -inner radius, m;  $R_2$ -outer radius, m), arising from a state of rest, is written in a cylindrical coordinate system as follows:

$$\rho \cdot \frac{\partial v_z}{\partial t} = - \frac{\partial P}{\partial z} + \frac{1}{r} \cdot \frac{\partial(r \cdot T_{rz})}{\partial r}; \frac{\partial T_{rz,k}}{\partial t} + \frac{\varphi_k}{\lambda_k} \cdot T_{rz,k} = \frac{\mu_k}{\lambda_k} \cdot \frac{\partial v_z}{\partial r};$$

$$\frac{\partial \mu_k}{\partial t} + \frac{\varphi_k}{\lambda_k} \cdot \mu_k = \frac{f_k}{\lambda_k}; k = 1, 2, \dots; T_{\tau z} = \sum_{k=1}^{\infty} T_{\tau z,k}; \Pi_D^2 = \left( \frac{\partial v_z}{\partial r} \right)^2. \tag{2}$$

Initial conditions for the system of equations (2):

$$T_{rz,k} = v_z = 0, t = 0. \tag{3}$$

“Adhesion” conditions are set on both surfaces. It is believed that the inner cylinder is maintained at a constant temperature  $T_1, \text{ }^\circ\text{C}$ , and the outer cylinder at a temperature  $T_2, \text{ }^\circ\text{C}$ . A constant temperature distribution is established inside the gap:

$$\frac{\theta - T_1}{T_2 - T_1} = \frac{\ln(r/R_1)}{\ln(R_2/R_1)} \tag{4}$$

When  $T_1 = T_2$  we have the case of isothermal flow. For the non-isothermal case, it is assumed that the viscosity is written according to the law [20]:

$$\eta = B \cdot \exp \frac{Q}{R \cdot \theta} \tag{5}$$

where  $Q$  – activation energy of the flow process,  $J \cdot mol^{-1}$ ;  
 $R$  – universal gas constant,  $R = 8.31446261815 J \cdot (mol \cdot K)^{-1}$ ;  
 $B$  – pre-exponential factor,  $Pa \cdot s$ .

Using the principle of temperature-time superposition, we write the dependence of the maximum relaxation time in the spectrum on temperature in the form [20]:

$$\lambda(\theta) = \lambda_s \cdot a_t(\theta). \tag{6}$$

where  $a_t$  – coefficient determined by the formula  $a_t = \eta/\eta_s$ ;  
 $\lambda_s$  – maximum relaxation time of the elastic properties of the liquid at the reference temperature, s;  
 $\eta_s$  – viscosity at the reference temperature, Pa s, which is taken as the average  $T_s = 0.5 \cdot (T_1 + T_2)$ .

The behavior of the liquid is determined by the following parameters: the elasticity number  $El = \lambda \cdot \eta_0 / (\rho h^2)$ ; the Weissenberg number  $We = \lambda \cdot \gamma_0$ , the law of distribution of the relaxation spectrum, determined by the direct parameter  $\alpha$  (7) and the temperature difference in the transverse direction in the gap. Wherein  $\eta_0$  is the initial Newtonian viscosity, Pa·s;  $\gamma_0$  – shear rate scale;  $h$  – gap width, m;  $\alpha$  is an experimental number (numerical parameter) characteristic of each type of liquid.

$$\lambda_n = \frac{\lambda}{n^\alpha}, n = 1, 2, 3, \dots, \infty \tag{7}$$

where  $\lambda$  – maximum relaxation time of the elastic properties of the liquid at a given temperature, s.

The study of the resulting system of equations can only be carried out numerically. To solve the problem, we use the finite difference method; Let us construct an implicit conservative system of difference equations, which can be obtained by the integro-interpolation method [25]. The influence of various rheological factors, such as the elasticity number  $El$ , the Weissenberg number  $We$ , the  $\alpha$  number, the number of relaxers in the spectrum on the flow, as well as the influence of the gap width and temperature difference between the pipes were analyzed. The main results of the calculations are shown graphically (Fig. 1-3).

**Results and discussion**

A viscous Newtonian fluid is characterized by a gradual filling of the velocity profile reaching a stationary value, which is maximum for a given flow (Fig. 1). For a viscoelastic fluid, the process of reaching a stationary regime (coinciding with the flow of the Newtonian fluid with the same initial viscosity for the linear model  $f_k = g_k = 1$ ) becomes oscillating (Fig. 1). It can be distinguished into an initial stage with a rapidly changing tension profile and a quasi-stationary stage. The magnitude of these stages depends on the value of the elasticity number  $El$ .

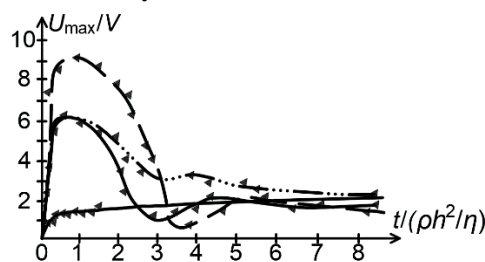


Fig. 1. Change in the maximum speed over time,  $\tau = t/(\rho h^2/\eta)$ : 1 – Newtonian fluid; 2 – linear viscoelastic fluid; liquid  $El = 10, \alpha = 2$ ; 3 – liquid at  $\alpha = 3$ ; 4 – Beard-Carreau model  $El = 10, We = 10, \alpha = 2, \delta = 0.1$ ; ▲ – experimental data from [7; 8; 20; 25]

Let us consider the quasi-stationary stage, when the inertia of the fluid is insignificant. It can be written in dimensional form:

$$I(t) \cdot \frac{\partial p}{\partial z} = \frac{1}{r} \cdot \frac{\partial}{\partial r} \left[ r \cdot \int_0^t \psi(t-t') \cdot \frac{\partial U(r,t')}{\partial r} \partial t' \right], \tag{8}$$

where  $\psi$  – relaxation function, Pa;  
 $I(t)$  – function creep, it is a dimensionless quantity;  
 $U$  – speed of fluid movement in the pipe, m/s.

Integrating expression (8) over  $r$ , and then taking the dimensional velocity profile in the form:

$$U(r,t) = \frac{1}{4} \cdot A(t) \cdot \left[ r^2 - R_1^2 - \frac{(R_1^2 - R_2^2)}{\ln(R_1/R_2)} \cdot \ln(r/R_1) \right] \tag{9}$$

using the Laplace transform, we can determine:

$$U(r,t) = \frac{1}{4} \frac{\partial I}{\partial t} \cdot \frac{\partial p}{\partial z} \cdot \left[ r^2 - R_1^2 - \frac{(R_1^2 - R_2^2)}{\ln(R_1/R_2)} \cdot \ln(r/R_1) \right] \tag{10}$$

for the Oldroyd model, in particular,

$$\psi = \frac{\eta}{\lambda_1} \cdot (1 - \beta) \cdot \exp\left(-\frac{t}{\lambda_1}\right) + \beta \cdot \lambda_1 \cdot \delta(t), \beta = \frac{\lambda_2}{\lambda_1}, \tag{11}$$

$$I(t) = \frac{t}{\eta} + \frac{\lambda_1 \cdot (1 - \beta)}{\eta} \cdot \left[ 1 - \exp\left(-\frac{t}{\beta \lambda_1}\right) \right].$$

where  $\beta$  – characterizes the ratio of the longest relaxation time, respectively, at the pipe radius  $R_1$  at temperature  $T_1$  and, respectively, at the pipe radius  $R_2$  at temperature  $T_2$ , and it is a dimensionless quantity;  
 $\delta$  – characterizes the growth rate of the thickness of the liquid layer within the pipe radius  $R_1$  in the longitudinal direction (along the axis of the pipe of the radius  $R_1$ ), and it is a dimensionless quantity.

Thus, we can write in dimensionless form:

$$u = \frac{1}{4} \cdot \frac{dI}{dt} \cdot \frac{\partial p}{\partial z} \cdot \left[ \frac{2y}{\delta} + y^2 - \frac{2 \cdot \ln(1 + \delta y)}{\delta \cdot \ln(1 + \delta)} - \frac{\ln(1 + \delta y)}{\ln(1 + \delta)} \right] \tag{12}$$

$$\frac{dI}{dt} = 1 + \frac{(1 + \beta)}{\beta} \cdot \exp(-t/(\beta \cdot El)).$$

Wherein  $r = R_1 \cdot (1 + \delta y)$ ,  $\frac{\partial p}{\partial z} = \frac{h^2}{\mu \cdot V} \cdot \frac{\partial p}{\partial z}$ ,  $u = \frac{U}{V}$ ,  $V = h^2 \cdot (\frac{\partial p}{\partial z})/\eta$ .

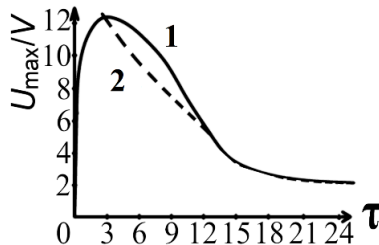


Fig. 2. **Maximum flow speed:** 1-result of numerical calculation;  
 2 – result of calculation using formula (12)

Fig. 2 shows graphs of changes in the flow velocity at the maximum point ( $\delta = 0.1$ ;  $\alpha = 4$ ;  $\lambda = 0.4898$ ;  $\beta = 0.076$ ;  $1 - \beta = 0.924$ ;  $El = 100$ ), obtained numerically for two relaxation times and according to formula (12). It is shown that, starting from the moment of time  $\tau = t/((\rho h^2)/\eta) = 15$ , the unsteady flow of a rheologically complex fluid can be described analytically using the relationships

given in this article. The difference between the obtained analytical results and the values of numerical calculations becomes less than 1%.

A study of the influence of the taken into account number of relaxers in the spectrum on the flow pattern is presented. Numerous calculations show that when solving problems, it is enough to use seven to ten terms in infinite series.

An increase in the parameter  $\alpha$  leads to an increase in the amplitude and to an increase in the number of oscillations (Fig. 1). As for the Newtonian fluid, a change in the thickness of the annular gap leads to a shift in the maximum value of the velocity: with an increase in  $\delta$ ,  $V_{\max}$  shifts towards the inner cylinder.

The results obtained are in qualitative agreement with the experimental studies [7; 8; 20; 25].

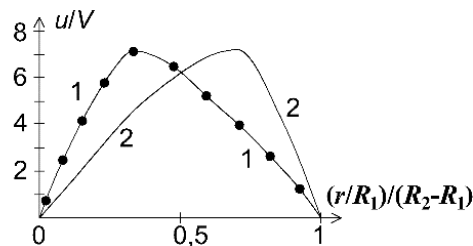


Fig. 3. Variation of velocity across the cross section during isothermal flow, Beard-Carreau model  $\delta = 0,1$ ,  $El = 10$ ;  $We = 10$ ,  $(R_1 \leq r \leq R_2)$ ,  $\tau = \frac{t}{(\rho h^2/\eta)} = 1:1$   $(T_1 - T_2)/(T_1 + T_2) = 0.125$ ,  $T_1 > T_2$ ; 2)  $(T_1 - T_2)/(T_1 + T_2) = -0.125$ ,  $T_2 > T_1$ .

Fig. 3 illustrates the influence of a constant temperature field across the gap on the flow development for the Beard-Carreau model. The process of the flow reaching a stationary regime is a wave one, and since the viscosity decreases with increasing the temperature, the maximum velocity shifts to a more heated wall.

The above numerical analysis showed that the rheological and thermophysical properties of the fluid have a significant impact on the development of the pressure flow of the viscoelastic fluid in the annular channel of the ground heat accumulator. These research results will help avoid problems with “clogging” the pipeline section with the mixture and installing additional pumps in the system at the design stage. And as a result, provide a reduction in operating energy and costs for pumping non-Newtonian liquids. The error in calculations and analytical calculations ranges from 5 to 10 percent, depending on the time range in which the fluid flow is studied. Moreover, the calculation results are stabilized at a small distance from the entrance to the pipeline, amounting to 3-5 pipeline radii.

## Conclusions

1. A physical-mechanical model of the non-isothermal flow of elastic-viscous liquids under the influence of a pulsed pressure gradient in the annular channels of ground heat accumulators is substantiated. The following are taken as a rheological model: 1) nonlinear integral model with a constancy function, depending on the invariants of the shear strain rate tensor; 2) Newtonian fluid; 3) viscoelastic fluid model; 4) Beard-Carreau model; 5) Meister model; 6) Macdonald-Beard-Carreau model; 7) Oldroyd model.
2. Numerical analysis of the resulting system of equations was carried out using the finite difference method. An implicit conservative system of difference equations was constructed, obtained by the integro-interpolation method. It has been established that the rheological and thermophysical properties of the fluid have a significant impact on the development of the pressure flow of elastic-viscous fluid in the annular channel. The patterns of the fluid flow and behavior are determined by two main parameters: a) elasticity number; b) Weissenberg number.
3. The results obtained in the work can be used in the future to calculate the main parameters of the flow of viscoelastic liquids in channels of various shapes and for various designs of heat accumulators (both for cases of isothermal flow and non-isothermal flow), operating in stationary and non-stationary (transient) modes.
4. The prospect of further research is to conduct experimental studies of the non-isothermal flow of elastic-viscous fluids under the action of a pulsed applied pressure gradient in the channels of ground heat accumulators.

### Author contributions

Conceptualization, Y.C., methodology, A.M. and S.R., software, Y.C., validation, S.R. and P.Z., formal analysis, Y.C. and A.M., investigation, Y.C., A.M., S.R. and P.Z., data curation, Y.C., A.M. and S.R., writing – original draft preparation, Y.C., writing – review and editing, A.M. and P.Z., visualization, S.R. and P.Z., project administration, A.M., funding acquisition, P.Z. All authors have read and agreed to the published version of the manuscript.

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